SMALL WORLD PROPERTIES OF A PERTURBED RAILWAY TIMETABLE.

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INTRODUCTION

The railway circulation and the sequence of trains on railway lines are regulated by severe rules as such as signal security systems, safety distance blocks and scheduled and delayed timetables. Timetable structures and the above rules are some of the aspects that influence capacity, but also regularity and reliability of the railway network.

Nowadays, analysis on stability of timetables have been conducted in two different directions: the one related with simulation analysis and optimization of the traffic and the one referred to analytical approaches on infrastructure and timetable characteristics. Therefore, a *Small World* weighted network approach, applied to a 'Timetable Service Network' (TSN) and not only to the topological view of the system, represents a new possible view to estimate the robustness of the timetable, seen as a type of complex weighted network.

A weighted network is a particular kind of graph structure in which every edge has been associated to a number that could represent the more various characteristics of the structure, the amount of seats available in flights over a route, or the information exchanged on a computer network.

It is hypothesized by some researchers (e.g. Barabási 2002) that the prevalence of *Small World* properties may reflect an evolutionary advantage in such network systems: one possibility is that *Small World* networks are more robust to perturbations than other network architectures.

In this paper, after a briefly review of existing literature about main important railway research projects and *Small World* properties tied up with this study, a generalized approach on *Small World* weighted network analysis will be presented in order to analyse railway timetables as a particular kind of network, with his own properties; after defining the used characteristics, the approach to solve a new kind of network, called Timetable Service Network (TSN) will be presented on a theoretical case of study as well as the simulation tool adopted to calculate the effects of perturbations on the network. General conclusions are going to complete this part of the study.

LITERATURE REVIEW

Literature in *Small World* networks properties is very large in number and it has been applied on several types of networks (biological, social, gene network, information) including railway metropolitan networks (e.g. Latora and Marchiori 2002). Moreover (e.g. Sen et al. 2002) an analysis of the *Small World* topological properties of the Indian Railway Network (IRN) has been developed to introduce the concept of "link" between two nodes as the connection derived from the service of a train which stops at any two stations (e.g. Seaton and Hackett 2003).

Analysis on weighted networks could be find in several studies (e.g. Barrat et al. 2004), and also in some transportation cases, such as an airport network, in which, as weight of edges, the available seats on flights during a fixed time period (usually an year) have been considered. A deterministic weighted network has been developed (e.g. Dorogovtsev and Mendes 2004) with a pseudo fractal approach, to take into account the evolution in time of topology and weights of links and vertices. *Small World* networks have been defined as well as classes of random graph (e.g. Watts and Strogatz 1998). In order to develop this study, another approach (e.g. Kurant and

Thiran 2006) has been considered; it takes into account three different types of spaces (changes, stops and stations) to obtain different results on the study of networks derived from timetable data (e.g. Liebers 2001).

Finally, weighted network analysis related to *Small World* properties have been developed (e.g. Latora and Marchiori 2003) in order to compare topological and weighted structures by means of an index called Efficiency, which will be described later in the paper, applied and modified to this study's topic.

PURPOSE OF THE STUDY

Small World networks seems to be more stable than other types of complex networks; their behaviour could be described between order and randomness conditions. This study aims to analyse the railway timetable robustness and reliability referred to a scheduled timetable and to a perturbed one affected by various kinds of failures, by means of intensive synchronous microsimulation model of the network.

Services, as said before, could be delayed because of several causes: this new approach tries to define a new perturbed network by uncertainly travel times, to analyse the stability of a complex weighted network compared with typical topological *Small World* properties by means of an extended definition of time-based network characteristics. The Timetable Service Network tries to help the analysis of a new level of robustness of the timetable, related with network stability.

COMPLEX WEIGHTED NETWORK

Network structures arise in very different contexts and situations: transportations cases are only a little part of a very large number of examples, such as social interactions networks, biological systems, information grids.

There are two common features that make different kind of networks *Small World*: the Characteristic Path Length L, measuring the typical separation between two generic nodes in the network (defined as the length, in number of edges, of the shortest path between two any vertices) and the Clustering Coefficient C, measuring the average cliquishness of a node. *Small World* networks are highly clustered, like regular lattices, while having small Characteristic Path Length, like random graphs.

Real networks, as railway services, show very heterogeneous characteristics in their behaviour, as well as different capacity and intensity of the connections, and therefore of the links' weights. They are also represented not only by the topological view of the system, but also by their dynamic behaviour; the characteristics of the connections as frequency of the services, travel times and delays are typical railway dynamical aspects to be analysed in a weighted network approach.

In the mathematics formalism (e.g. Watts and Strogatz 1998), a generic network is represented by a graph $\mathbf{G}$ with N nodes and K edges between nodes. A graph can be expressed by means of its adjacency matrix $\mathbf{A}(a_{ij})$, whose elements take the value 1 if an edge connects vertices i and j, 0 otherwise. The main features of complex weighted systems are shown in the next paragraphs.

Weight And Vertex Strength

Weighted networks are usually described by a matrix $\mathbf{W}(w_{ij})$, whose elements specifies the weight of the edge connecting vertices i and j. A significant measure of complex network has been derived extending the definition of vertex degree (k_i) in terms of vertex strength s_i, defined as follows (e.g. Barrat et al. 2006):

$$s_i = \sum_{j=1}^{N} a_{ij} w_{ij} \quad ;$$

It is useful to remind the definition of 'vertex degree': in graph theory the degree of a node represents the number of its connections towards the others in the network; as it will be shown later in the paper, in this context two nodes have got a connection if there is at least one service connecting each other.

Weighted Clustering Coefficient

This is a typical measure derived from the topology, with the main difference that in this case the weighted coefficient of clustering takes into account the different weights and the behaviour of links, in particular the connections between the neighbours nodes. Various kind of weighted Clustering Coefficients have been developed in the scientific literature (e.g. Saramäki et al. 2006); in this study the following approach has been considered (e.g. Onnela et al. 2005) because it takes into account all weights of all edges in triangles (three nodes connected to each other make a triangle), and because of its purely weight-based structure. Its range is between 0 and 1 and has been defined as follows:

$$C_{i,0} = \frac{1}{(k_i(k_i-1))} \sum_{j,h} (\hat{w}_{ij} \hat{w}_{ih} \hat{w}_{jh})^{1/3} \quad ;$$

The weights $\hat{w}$ are normalised by the maximum weight in the network. Defining the global quantity C^w as the average value of weighted clustering over all vertices, it will be possible to provide global information about the correlations between weights and topology. In the case of random networks $C^w = C_R$ (Clustering Coefficient related to random networks) but in the case of real weighted networks $C^w > C_R$ or $C^w < C_R$.

Efficiency

This measure has been developed (e.g. Latora and Marchiori 2003) as a natural extension of the *Small World* definition for weighted systems. The Efficiency E measures how efficiently the nodes exchange information. In this case, information is related with the edges' weights, and therefore with the uncertain travel times and timetable structures' characteristics; the more the edge weights, the less efficient it is.

Taking into account the weight of each length, Efficiency can be defined as $\varepsilon_{ij} = 1/\hat{w}^a_{ij}$ where $\hat{w}^a_{ij}$ is calculated both by the weighted $\mathbf{W}(w_{ij})$ and the adjacency $\mathbf{A}(a_{ij})$ matrices. The global and local quantities associated with the graph allow the system to predict the *Small World* behaviour of a weighted complex network. The average Efficiency of the network can be defined as follows, while later in the paper will be presented the the quantities to be calculated in order to compare topological and weighted *Small World* properties:

$$E = \frac{1}{(N(N-1))} \sum_{i \neq j \in G} \epsilon_{ij} \quad ;$$

THE TIMETABLE SERVICE NETWORK (TSN): A NEW TIME-BASED APPROACH

After defining the main characteristics of complex networks, the following analysis demands that, in the evaluation of the parameters, both the "travel time" of the services and the variation of headways appear as a factor, in order to predict the behaviour of the Timetable Service Network under perturbed conditions. This means that the weight of each edge (service) and of each node (station) has to be defined. According to previous studies on complex networks, more general approaches could increase the possibility to elaborate the *Small World* analysis to almost any kind of complex network (e.g. De Montis et al. 2006). The so called Timetable Service Network (TSN) is the new kind of graph structure that analyse both the weighted characteristics and the topology of different spaces of services in a railway network.

Before defining the weighted model network, the following paragraphs introduces the topological model developed and its timetable structure.

Topological Model Network and Timetable Services

The following analysis have been supported by a microsimulation software (e.g. Nash and Huerlimann 2003) which simulates the run of trains on the modelled network, their conflicts, various kind of block systems and all the remaining railway circulation characteristics.

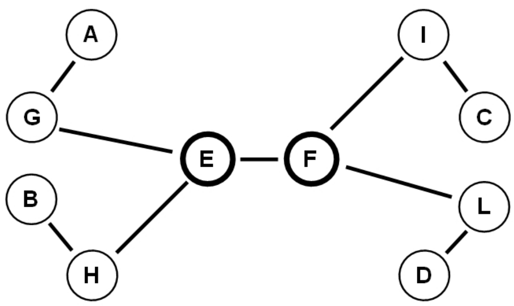

Figure 1 Topological Model Network

The modelled network consists of four double-track lines (Figure 1**)** of about 30 kilometres each of them converging on a single double-track line of the same length. Services alternate between intercity and regional trains from all terminal stations. Both intercity and regional trains stop at terminal and at the two common stations, while regional trains stop at all stations. An artificial timetable has been built on a simple network to provide a model and output data to calculate the parameters of the circulation. The timetable presents a symmetrical periodic passenger timetable with four trains/hour in the common branch; meanwhile other branches of the networks are covered by two trains/hour, in both directions.

In this first analysis, perturbed conditions on the network have been defined in term of "mean departure delays" for services: the simulator defines an average given beginning delay for courses without initial delay. Mean delays have been defined in three different scenarios of 300, 600 and 900 seconds; this fact not means that all services are delayed equally in each scenario, because the distribution of delays is different with different initial delay time.

After simulating all scenarios, timetable data have been exported in an external application in order to evaluate the different characteristics of the circulation.

Modelling the Weighted Network

The following approach derives from the analysis on timetable developed by means of different space networks (e.g. Kurant and Thiran 2006). Figure 2 is related to the Timetable Service Network scheme made by the "space of stops" definition: 'two stations are connected if they are two consecutive stops on a route of at least one vehicle'. Nodes are characterized by every departure (in every stop station) of a service and therefore by their headways, while edges are related on the variable travel times between nodes. The different gradient of the nodes and of the edges is directly connected with the weight and the strength measures.

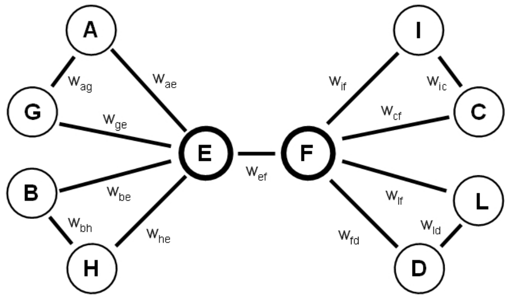

Figure 2 Timetable Service Network

Definition of Weights

Each edge in the previous diagram is associated at a weight w_{ij}; this measure has been defined by means of two different elements: the first, called "Compactness"

(C_p) (e.g. Lamanna et al. 2006) takes into account the variability of the headways a_i with increasing mean delays in two general sections of a line. M_a and n are respectively the average value of the headways values and the number of trains circulating on the line and it has been defined as follows:

$$C_P = \frac{\sqrt{(a_1 - M_a)^2 + \ldots + (a_n - M_a)^2 / n}}{M_a \sqrt{n-1}} \quad ;$$

This measure allows the system to consider the structure of the timetable and its variations in case of perturbed conditions or in case of failures. The second index is called “Performance” (*P*), and considers the amount of delayed services in different delay scenarios, defined by the ratio between the delayed headway time and the scheduled one, in every scenario. C_p range goes from 0 (minimum compactness and equal distribution of trains) to 1 (maximum compactness), while P value is 1 in case of perfect schedule situation, and is increasing with delays scenarios.

Compactness and Performance have been defined on both directions of travel, in order to model a directional weighted network. By means of these two measures, it has been possible to define the weight of each link and to provide values of all the elements characterizing the Timetable Service Network: the strength of each node, the weighted Clustering Coefficient and the time-based Efficiency.

RESULTS

The Timetable Service Network (TSN) results are shown in two different paragraphs: the first one presents the evaluation of the “space of stops” topological network properties by means of typical indices as well as Characteristic Path Length and Clustering Coefficient, to verify if this network could be defined as *Small World*; the second one is related to the weighted system, and to the correlations with the topological one, by means of Efficiency measures.

TSN: Topological Properties

Many studies have been developed in order to measure the topological properties of a network. The *Small World* properties of the “space of stops” network have to be verified by the evaluation of the Characteristic Path Length *L* and the Clustering Coefficient *C*, and by their comparison to the corresponding ones of a random network (L_R and C_R). In fact, a network could be defined as *Small World* if *L* is comparable to that of a random graph, while it displaying significantly more clustering, like regular structures.

In random topological networks, the Clustering Coefficient and the Characteristic Path Length are relative small compared to the ones related with regular networks, and have been defined as follows (e.g. Watts and Strogatz 1998), where *Z* is the mean value of the vertices degree and *N* the number of nodes:

$$C_R = Z/N \quad ; \quad L_R = \ln N / \ln Z \quad ;$$

On the other side, non-random networks could be defined as structures having regular behaviour like the so called physics 'lattices' networks. Regular networks have got high values of both Clustering Coefficient and Characteristic Path Length. By definition, the behaviour of *Small World* networks is nor random neither completely ordered; it is something between them. The results show that the Characteristic Path Length (L = 2.06) is lower but comparable to the random one (L_R = 2.41), while the Clustering Coefficient (C = 0.84) is higher than the one related to random networks (C_R = 0.26). This means that the 'space of stops' network has got some of the

Small World properties, that allow a user to travel from one node to another one in a relative small number of steps.

TSN: Weighted and Efficiency Properties

After defining the topological properties of the network, the analysis on the weighted one will be presented, looking at the different delay scenarios and at their correlations with *Small World* properties. The following results show the differences of various characteristics of the TSN in every single scenario: nodes strength, edges weight and weighted Clustering Coefficient.

Figure 3 is related to the weights calculated in the four scenarios, including the scheduled starting case, for each direction of travel. As it was supposed to be, increasing delays correspond to increasing edge weights; the little variations in the edge weights (dir.*ij*) for I-C branch and (dir.*ji*) for F-C, I-C and F-I branches are due to the simulator's delay distribution; different scenarios are related to different distributions of perturbed conditions. In this test case, it has been used the same scenario to compare the same timetable under the same kind of failures, modified only in the mean delay time.

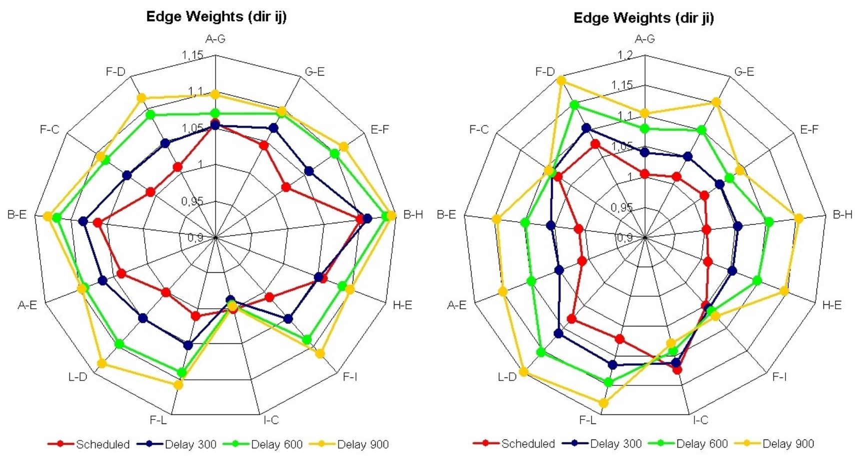

Figure 3 Edges Weights of Both Directions of Travel

Figure 4 shows the strength behaviour of the nodes in each delay scenario, and in the scheduled one. It has been reported only one diagram related with one direction (dir.*ij*) because there are only few differences with the other direction one. The 'Hub' behaviour of E and F nodes is very clear despite the few changes of this measure with increasing mean delays. This means that the railway timetable is stable around its Hubs, and its behaviour remains almost constant with increasing perturbed conditions.

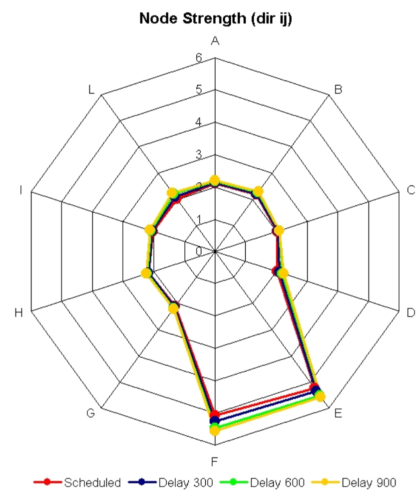

Figure 4 Nodes Strength

Figure 5 shows the course of the weighted Clustering Coefficients for each node. The dotted line represent the average value of the weighted Clustering Coefficient (C_w) calculated on all scenarios and compared with the one related to random networks C_R (black continuous line). The value of C_w is always higher than C_R in all delay scenarios. This means that the network is built by interconnected triple of vertices, formed by edges with large weight. This result also reveals the existence of a particular phenomenon in which important stations (such as E and F) form a group of well interconnected nodes. Therefore the timetable behaviour under perturbed conditions depends on the hubs' characteristics, on their headway compactness, and on how a delayed timetable structure is modified by these properties (as capacity and bottleneck phenomenon could be). In fact, the low values of the weighted Clustering Coefficient at nodes E and F means that there are only few connections between the stations connected to the hubs; all services have to pass through these two main stations to reach their other destinations.

Finally, it is interesting evaluating the Efficiency of the network in three delay scenarios, in order to compare the behaviour of the network both by topological and weighted *Small World* properties. shows the results on this study's topic. E_{glob} is the value of E normalized in [0,1] range by the Efficiency value in the ideal case, in which information is delivered in the most efficient way. E_{loc} is the same of the previous measure related to the subgraph of the neighbours of every single node. High values of E_{glob} and E_{loc} mean that the weighted network could be seen as *Small World*, in terms of Efficiency. Results are able to define the variation of Efficiency in communications between nodes; in this railway timetable case, it means that the TSN is almost 3% less efficient than an ideal network (in the scheduled case), where every link presents the minimum value of compactness and a scheduled travel time. The global Efficiency decreases with increasing mean delays, as expected, along the delay scenarios. On the other hand, relative low values of E_{loc} indicate a poor local Efficiency; therefore a failure in a station should affect the Efficiency on all over the network. This topic could be useful to predict the behaviour of the nodes under perturbed conditions, and their Efficiency related both to local and to global network performance.

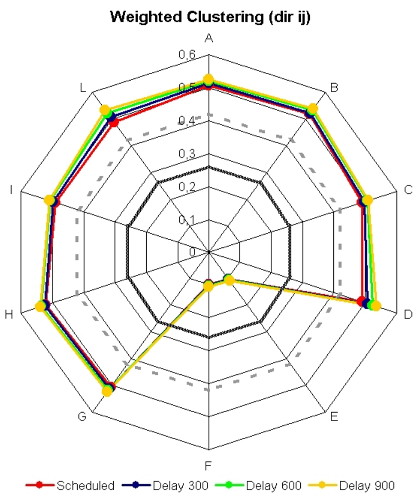

Figure 5 Weighted Clustering Coefficient

Table 1 Efficiency Measures

SCENARIOS	E_{glob}		E_{loc}	
	dir_{ij}	dir_{ji}	dir_{ij}	dir_{ji}
scheduled	0,97	0,12	0,96	0,12
delay300	0,95	0,12	0,93	0,11
delay600	0,92	0,11	0,91	0,11
delay900	0,91	0,10	0,88	0,10

CONCLUSIONS AND FURTHER RESEARCH ANALYSIS

In this study a new approach on railway timetables stability has been developed; its behaviour under perturbed conditions is described by some measures applied on this study's topic. The *Small World* behaviour of the 'space of stops' network has been compared to the new time-based weighted network in order to get new information about hubs' behaviour, their connectivity, stations strength and general stability properties related to the well-known *Small World* network solidity. The *Small World* behaviour topological properties of the TSN seem to reflect a very stable behaviour of the system, and more accurate analysis on the weighted TSN indicate that its timetable's structure has got only some of the founded stability and efficiency characteristics. Therefore the topology and the timetable of a network have to be investigated as a single entity to predict the real behaviour of the system.

The research is now directed to a further extended weight and Efficiency definitions, to take into account more railway timetables aspects as heterogeneity (e.g. Vromans 2005) and some real delay statistics to be included in the simulations; some further indications about the travel demand (occupation rate of the services) could be included in the weight function in order to consider the amount of real traffic pattern.

REFERENCES

[1]. Barabási A.-L. (2002) Linked: The New Science of Networks. Perseus, Cambridge, MA, USA.

[2]. Barrat A., Barthélemy M., Pastor-Satorras R., Vespignani A. (2004) The architecture of complex weighted networks. Proc. Natl. Acad. Sci. USA, 101(11):3747.

[3]. De Montis A., Barthélemy M., Chessa A., Vespignani A. (2006) The structure of Inter-Urban traffic: a weighted network analysis. physics/0507106 v2.

[4]. Dorogovtsev S.N., Mendes J.F.F. (2004) Minimal models of weighted scale-free networks. cond-mat/0408343 v2.

[5]. Kurant M., Thiran P. (2006) Trainspotting: extraction and analysis of traffic and topologies of transportation networks. physics/0510151 v2.

[6]. Lamanna F., Longo G., Santorini F. (2006) Perturbed timetable structure analysis and railway circulation quality: a test case. Proceedings of the International Symposium of Transport Simulation 2006, EPFL, Lausanne, Switzerland.

[7]. Latora V., Marchiori M. (2002) Is the Boston subway a *Small World* network? Physica A 314, pp. 109-113.

[8]. Latora V., Marchiori M. (2003) Economic *Small World* behaviour in weighted networks. Eur. Phys. J. B 32, 249-263.

[9]. Liebers A. (2001) Analyzing Train Time Table Graphs, PhD Thesis. Dep. of Computer and Information Science. University of Konstanz, Germany.

[10]. Nash A., Huerlimann D. (2003) OPENTRACK - Simulation of railway networks, user manual version 1.3. Institute for Transportation Planning and Systems. ETH Zurich. Switzerland.

[11]. Onnela J.-P., Saramäki J., Kertész J., Kaski K. (2005) Intensity and coherence of motifs in weighted complex networks. Phys. Rev. E 71, 065103.

[12]. Saramäki J., Kivelä M., Onnela J-P., Kaski K., Kertész J. (2006) Generalizations of the *Clustering Coefficient* to weighted complex networks. cond-mat/0608670 v2.

[13]. Seaton K.A., Hackett L.M. (2003) Stations, trains and *Small World* networks. cond-mat/0311254 v1.

[14]. Sen P., Dasgupta S., Chatterjee A., Sreeram P.A., Mukherjee G., Manna S.S. (2002) *Small World* properties of the Indian Railway Network. cond-mat/0208535 v2.

[15]. Vromans M.J.C.M. (2005) Reliability of Railway Systems. PhD Thesis, Erasmus University of Rotterdam, Holland.

[16]. Watts D.J., Strogatz S.H. (1998) Collective dynamics of '*Small World*' networks. Nature, 393, pp. 440-442.